Firm-Destination Heterogeneity and the Distribution of Export Intensity*

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Abstract

We derive closed-form expressions for the distribution of export intensity when firm-destination-specific revenue shifters are distributed gamma, Fréchet and Weibull in a two-country model of trade with isoelastic demand. We estimate the parameters governing the distribution of export intensity for each type of revenue shifter across 72 countries. We compare the model's fit to the distribution of export intensity across countries when revenue shifters are distributed lognormal, gamma and Fréchet/Weibull. While lognormal slightly outperforms the other distributions, all revenue shifters considered reproduce salient features of export intensity distributions within and between countries quite successfully.

Keywords: Exports; Export Intensity Distribution; Firm Heterogeneity.
JEL classification: F12, F14, C12, O50.

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1 Introduction

Contrary to received wisdom (Bernard et al., 2007; Melitz and Redding, 2014), the distribution of export intensity—the share of exporters' sales accounted for by exports—varies substantially across countries and is often bimodal, with high shares of both low- and high-intensity exporters coexisting within a country. Defever and Riaño (2022) show that incorporating firm-destination-specific revenue shifters that are distributed lognormal into a standard two-country model of trade with isoelastic demand can account for most of the observed variation in the distribution of export intensity across countries.

In this paper we show that gamma, Fréchet and Weibull-distributed revenue shifters also reproduce salient features of export intensity distributions within and between countries quite successfully.[1] While the lognormal distribution is used more frequently in models of international trade with monopolistic competition, the Fréchet and gamma distributions feature prominently in Ricardian models in the spirit of Eaton and Kortum (2002) and in the dynamic model with imperfect imitation of Luttmer (2007) respectively, while Cherkashin et al. (2015) use Weibull-distributed demand shifters in a model of trade with monopolistic competition.

We carry out our analysis in two steps: first, we derive the probability density function (pdf) of export intensity for each distribution of revenue shifters and estimate its parameters by maximum likelihood across 72 countries. Secondly, we set up a horse-race between the models based on lognormal, gamma and Fréchet-distributed shifters in terms of their fit of export intensity distributions. We find that while lognormal slightly outperforms the other distributions in terms of the number of countries for which it provides the best fit to the data, all types of revenue shifters considered reproduce the cross-country variation of export intensity distributions observed in the data quite well.

The choice of which probability distribution to use to model heterogeneity is a central one in international trade. An extensive literature has shown that the distribution of firm-specific productivity crucially shapes outcomes such as the welfare gains from trade, the trade elasticity and the extent of firm misallocation (Chaney, 2008; Head et al., 2014; Melitz and Redding, 2015; Nigai, 2017; Fernandes et al., 2018; Coughlin and Bandyopadhyay, 2020; Egger et al., 2020; Mrázová et al., 2021). This paper contributes to understand the implications of firm-destination-specific heterogeneity—a source of variation that is as important as productivity in explaining firm-level export sales (Kee and Krishna, 2008; Eaton et al., 2011; Munch and Nguyen, 2014).

2 Model

Consider a monopolistically-competitive industry in which each firm ω produces a unique, differentiated good. Firms can sell their output domestically (d) or abroad (x) and face an isoelastic

[1] As we show in Section 2 below, the distribution of export intensity that obtains when revenue shifters are distributed Weibull and Fréchet is the same because when X is distributed Weibull, X^{-1} follows a Fréchet distribution. Thus, from here on onwards we refer only to Fréchet shifters with the understanding that the same properties and estimates obtain if the underlying revenue shifters were distributed Weibull.

demand in each market. The optimal price in each market i is the standard constant markup over marginal cost and therefore, sales, $r_i(\omega)$, can be written as:

$$r_i(\omega) = s_i \cdot \Phi(\omega) \cdot z_i(\omega), \quad i \in \{d, x\}. \tag{1}$$

s_i collects variables common to all firms selling in market i (aggregate income and price index in the destination market, the wage in the origin's country); $\Phi(\omega)$ varies across firms but not destinations (productivity) and $z_i(\omega)$ is a firm-destination-specific term that can accommodate, among other things, differences in preferences across countries (Crozet et al., 2012), or the availability of policies that incentivize firms to export instead of selling domestically (Brooks and Wang, 2016; Defever and Riaño, 2017; Defever et al., 2019).

Firms incur a fixed cost f_i to operate in market i. We assume that these are paid after observing productivity, but before revenue shifters are realized. This ensures that firms' export intensity is unaffected by the distribution of productivity and, in turn, allows us to derive closed-form expressions for the pdf of export intensity.[2]

Export intensity, $E(\omega)$, is the share of exporter ω's sales accounted for by exports, and e denotes a specific realization of this random variable. Thus,

$$E(\omega) \equiv \frac{r_x(\omega)}{r_d(\omega) + r_x(\omega)} = \frac{s_x z_x(\omega)}{s_d z_d(\omega) + s_x z_x(\omega)} = \frac{Z_x(\omega)}{Z_d(\omega) + Z_x(\omega)}, \tag{2}$$

where $Z_i(\omega) \equiv s_i z_i(\omega)$. We use the method of transformations to derive the pdf of export intensity when revenue shifters are independent across destinations and are distributed gamma and Fréchet. Note that if revenue shifters Z_i are independent Weibull, then Z_i^{-1} follows a Fréchet distribution with the same scale and shape parameters, and therefore, Z_x/Z_d is a ratio of independent Fréchet random variables. As a result, the distribution of export intensity when revenue shifters are Weibull is the same as when the underlying shifters are Fréchet.

Our first proposition reads:

Proposition 1. *Assume firm-destination-specific revenue shifters $\{z_i(\omega)\}_{i\in\{d,x\}}$ are drawn from the same distribution independently across destinations.*

(i) When revenue shifters are distributed gamma (Γ) with scale parameter 1 and shape parameter $\alpha > 0$, i.e. $z_i(\omega) \sim \Gamma(1, \alpha)$, and therefore, $Z_i(\omega) \equiv s_i z_i(\omega) \sim \Gamma(s_i, \alpha)$, then, the pdf of export intensity is given by:

$$h^{\Gamma}(e) = \frac{\left(\frac{s_d}{s_x}\right)^{\alpha}}{\mathcal{B}(\alpha, \alpha)} \times \frac{e^{\alpha-1}(1-e)^{-(1+\alpha)}}{\left[1 + \left(\frac{s_d}{s_x}\right)\left(\frac{e}{1-e}\right)\right]^{2\alpha}}, \quad e \in (0, 1), \tag{3}$$

where $\mathcal{B}(\cdot, \cdot)$ denotes the Beta function

[2] If firms made their operation decision based on the realization of productivity and revenue shifters instead, then the truncation in the distribution of firms' sales in a given market induced by the fixed cost would prevent us from using the method of transformations of random variables to obtain the pdf of export intensity.

(ii) *When revenue shifters are distributed Fréchet (Weibull) with scale parameter 1 and shape parameter* $\alpha > 0$, *i.e. when* $z_i(\omega) \sim$ *Fréchet* $(1, \alpha)$, *and therefore,* $Z_i(\omega) \equiv s_i z_i(\omega) \sim$ *Fréchet (Weibull)* (s_i, α), *then, the pdf of export intensity is given by:*

$$h^{\text{Fréchet}}(e) = \alpha \left(\frac{s_d}{s_x} \right)^{\alpha} \times \frac{e^{\alpha - 1}(1-e)^{-(1+\alpha)}}{\left[1 + \left(\frac{s_d}{s_x} \right)^{\alpha} \left(\frac{e}{1-e} \right)^{\alpha} \right]^2}, \quad e \in (0,1). \tag{4}$$

Proof. See Appendix A.1.

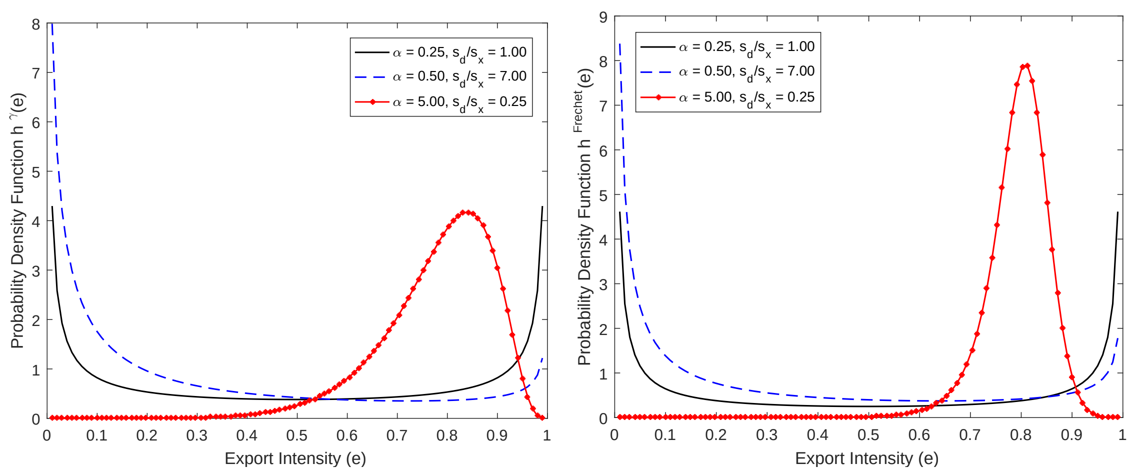
The conditions under which the distribution of export intensity is bimodal are spelled out in Proposition 2:

Proposition 2. *When revenue shifters are distributed gamma, Fréchet or Weibull and the shape parameter* α *is strictly lower than 1, then the distribution of export intensity is bimodal with modes located at 0 and 1. The major mode occurs at 0 when* $s_d/s_x > 1$, *and at 1 in the converse case; if* $s_d/s_x = 1$, *then the distribution is symmetric around 0.5.*

Proof. See Appendix A.2.

Figure 1 presents examples of the pdf of export intensity for the two distributions of revenue shifters and different parameter values. Although the critical value of the shape parameter necessary to produce bimodality is the same for gamma and Fréchet-distributed reveneue shifters, the expected value of shifters tends to infinite when $\alpha < 1$ for the latter; this does not happen when shifters follow gamma or lognormal distributions.

Figure 1: Pdf Export Intensity Distribution—Gamma and Fréchet-Distributed Revenue Shifters



Lastly, we show that we can recover a country's relative market size vis-à-vis the rest of the world, s_d/s_x, using the median export intensity:

Proposition 3. *If revenue shifters are independent across firms and destinations and follow gamma, Fréchet or Weibull distributions as specified in Proposition 1, then the median export intensity, e^{med}, is given by:*

$$e^{med} = \frac{s_x}{s_d + s_x}, \tag{5}$$

Proof. See Appendix A.3.

The gamma and Fréchet-based distributions of export intensity have similar properties to those derived under lognormal revenue shifters by Defever and Riaño (2022). Namely, the distributions are bimodal with modes near 0 and 1 when the dispersion of revenue shifters is sufficiently high and the location of the major and minor modes is determined by a country's relative market size. The latter, in turn, is pinned down by the median export intensity, as shown in (**??**) above.

3 Estimation

We use firm-level data for 72 countries gleaned from several waves of the World Bank Enterprise Surveys spanning the period 2002-2016 (Defever and Riaño, 2022). We recover each country's relative market size using its median export intensity, and conditional on it, estimate the shape parameters in (3) and (4) by maximum likelihood.[3]

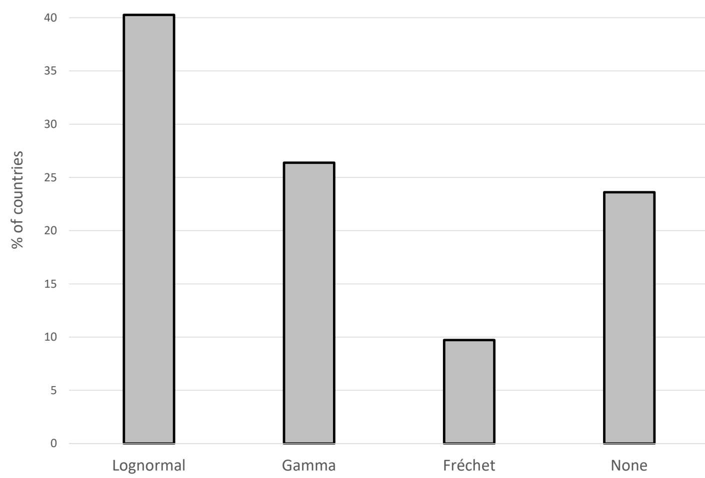
4 Results

Table 1 reports the relative market size and shape parameter for each country and distribution of revenue shifters. Relative market size varies substantially across countries, with the median country in the sample having an effective domestic market size that is 50% larger than its export market. The mean of shape parameters across countries is 0.672 for gamma and 0.740 for Fréchet-distributed revenue shifters. These estimates imply that the distribution of export intensity is bimodal in all but 6 and 7 countries when shifters are distributed gamma and Fréchet respectively.

The correlation between the predicted and observed share of exporters across 5 export intensity bins across all countries is close to 0.9 for lognormal, gamma and Fréchet revenue shifters. We use the Vuong (1989) test to carry out pairwise comparisons between the three types of revenue shifters in terms of their fit of the distribution of export intensity in each country. The Vuong test allows us to compare non-nested models obtained from different families of distributions in a directional way—thereby indicating which model fits the data better when we reject the null hypothesis that the two models are indistinguishable from each other. Table 2 reports the pairwise comparisons for each country. A positive number greater than 2.58 (the critical value at the 1% level) means that model 1 fits the data better than model 2; a negative number below the critical value indicates that model 2 outperforms model 1 instead; otherwise we cannot discriminate between the two models.

[3] We censor the export intensity of 'pure exporters' (i.e. those exporters that sell all their output abroad) at 0.99 because the export intensity pdfs (3) and (4) are not defined at an export intensity of 1. Increasing the censoring cutoff biases the value of the shape parameter downwards in the direction of bimodality.

Figure 2: Pairwise Comparison of Goodness-of-fit of Export Intensity Distribution



The figure plots the percentage of countries for which we can determine that a given distribution of revenue shifters dominates the other two alternatives based on the Vuong tests reported in Table 2 (e.g. if for a given country, lognormal revenue shifters provide a better fit than gamma and Fréchet ones at the 1% level). There are 10 countries for which gamma and Fréchet both provide a better fit to the data than lognormal, but we cannot discriminate between the two of them; we assign half of these to gamma and the other half to Fréchet.

Figure 2 summarizes our main result. Lognormal revenue shifters provide the best fit for the distribution of export intensity for the highest number of countries in our data (29 out of 72). Nevertheless, gamma or Fréchet shifters do better than lognormal in more than one-third of the countries, and in one-fourth of them no distribution clearly outperforms the other two. Our horse-race suggests that any of three distributions considered provides an excellent fit to the distribution of export intensity within and across countries.

5 Conclusion

In this paper we have derived closed-form expressions for the pdf of export intensity in a workhorse model of trade of two countries with isoelastic demand when firm-destination-specific heterogeneity in the form of market-specific revenue shifters is distributed gamma, Fréchet and Weibull. We show that the model reproduces the substantial variation and high prevalence of bimodality observed in the distribution of export intensity across countries when revenue shifters follow any of the aforementioned distributions.

We carry out a horse-race between models featuring gamma, Fréchet/Weibull and lognormal-distributed revenue shifters in terms of their ability to fit the distribution of export intensity across countries. We find that while lognormal slightly outperforms the other two, any of the these distributions reproduce salient features of export intensity distributions within and across countries quite successfully.

Table 1: Country-Specific Estimates of Relative Market Size and Shape Parameter

Distribution:		Gamma	Fréchet	Distribution:		Gamma	Fréchet
Parameter:	s_d/s_x	α	α	Parameter:	s_d/s_x	α	α
Country:	(1)	(2)	(3)	Country:	(4)	(5)	(6)
Albania	0.47	0.37	0.47	Lithuania	0.67	0.52	0.60
Argentina	5.67	1.05	1.04	Madagascar	0.01	0.44	0.53
Armenia	2.33	0.65	0.73	Malaysia	1.50	0.84	0.89
Bangladesh	0.01	0.88	1.06	Mauritius	0.82	0.43	0.51
Belarus	1.50	0.82	0.86	Mexico	2.33	0.80	0.85
Bolivia	2.08	0.62	0.70	Moldova	0.67	0.56	0.64
Bosnia & Herzegovina	2.33	0.62	0.70	Morocco	0.01	0.50	0.60
Brazil	9.00	0.76	0.85	Namibia	4.00	0.45	0.53
Bulgaria	0.67	0.54	0.62	Nicaragua	1.50	0.60	0.68
Chile	4.00	0.67	0.74	Nigeria	1.00	0.77	0.83
China	1.50	0.52	0.61	Pakistan	0.11	0.46	0.55
Colombia	4.00	1.06	1.04	Panama	2.57	0.62	0.71
Costa Rica	2.33	0.60	0.68	Paraguay	1.50	0.60	0.67
Croatia	1.50	0.63	0.71	Peru	2.33	0.56	0.64
Czech Rep.	1.00	0.79	0.84	Philippines	0.01	0.41	0.50
Ecuador	5.67	0.64	0.72	Poland	2.33	0.81	0.87
Egypt	2.33	0.73	0.80	Romania	0.25	0.54	0.62
El Salvador	1.50	0.56	0.64	Russian Fed.	9.00	1.12	1.11
Estonia	0.49	0.59	0.66	Senegal	2.64	0.71	0.79
Ethiopia	1.22	0.54	0.62	Serbia	4.00	1.20	1.15
FYR Macedonia	0.43	0.52	0.60	Slovak Rep.	1.35	0.66	0.73
Ghana	2.33	0.88	0.94	Slovenia	1.00	0.81	0.84
Guatemala	2.33	0.66	0.75	South Africa	5.67	1.21	1.17
Honduras	1.50	0.43	0.51	Sri Lanka	0.01	0.73	0.86
Hungary	1.50	0.68	0.75	Sweden	1.00	0.73	0.79
India	1.50	0.50	0.58	Syrian Arab Rep.	2.33	0.93	0.97
Indonesia	0.54	0.55	0.63	Tanzania	4.00	0.75	0.82
Ireland	1.50	0.48	0.57	Thailand	0.67	0.56	0.64
Jordan	0.69	0.59	0.66	Tunisia	0.43	0.42	0.51
Kazakhstan	4.00	0.92	0.97	Turkey	0.67	0.61	0.69
Kenya	2.33	0.67	0.75	Uganda	2.33	0.63	0.71
Korea, Rep.	1.86	0.73	0.79	Ukraine	2.33	0.88	0.93
Kyrgyz Rep.	1.00	0.74	0.80	Uruguay	1.56	0.57	0.65
Lao PDR	0.21	0.60	0.68	Uzbekistan	2.33	0.62	0.70
Latvia	0.43	0.50	0.59	Vietnam	0.25	0.51	0.59
Lebanon	2.33	1.00	0.99	Zambia	4.00	0.72	0.81

Coefficients estimated by maximum likelihood, conditional on $s_d/s_x = (1/e^{med}) - 1$. All estimated shape parameters are statistically different from 0 at the 1% level.

Table 2: Paiwise Tests of Goodness-of-fit for Different Distributions of Revenue Shifters

Model 1:	$\mathcal{LN}$	$\mathcal{LN}$	Γ	Model 1:	$\mathcal{LN}$	$\mathcal{LN}$	Γ
Model 2:	Γ	Fréchet	Fréchet	Model 2:	Γ	Fréchet	Fréchet
Country:	(1)	(2)	(3)	Country:	(4)	(5)	(6)
Albania	15.87	27.67	-11.29	Lithuania	9.46	13.47	-6.42
Argentina	-5.48	-5.44	-1.29	Madagascar	-3.97	-2.54	6.54
Armenia	-0.95	-0.87	1.12	Malaysia	-4.34	-4.04	3.48
Bangladesh	-21.44	-11.62	0.47	Mauritius	11.59	18.73	-8.08
Belarus	0.79	0.76	-0.89	Mexico	-4.47	-4.51	2.73
Bolivia	1.95	1.93	-1.98	Moldova	7.47	10.35	-4.93
Bosnia & Herzegovina	0.25	0.10	-0.58	Morocco	-9.96	-5.96	18.95
Brazil	-14.05	-12.56	4.13	Namibia	1.74	1.95	-1.44
Bulgaria	10.49	14.41	-7.03	Nicaragua	0.36	0.40	-0.25
Chile	-0.13	-0.76	-1.74	Nigeria	-3.45	-2.72	5.32
China	7.07	7.96	-5.66	Pakistan	27.39	21.37	-27.51
Colombia	-2.88	-2.80	-0.36	Panama	-1.11	-0.92	1.58
Costa Rica	0.68	0.60	-0.83	Paraguay	4.11	4.72	-3.25
Croatia	2.52	2.67	-2.19	Peru	2.71	2.88	-2.37
Czech Rep.	0.87	0.81	-1.07	Philippines	-5.03	-2.58	8.72
Ecuador	-3.43	-3.44	2.52	Poland	-2.88	-2.95	1.77
Egypt	-5.54	-4.86	5.45	Romania	8.90	10.55	-6.69
El Salvador	2.61	3.15	-1.57	Russian Fed.	-4.87	-4.92	-2.00
Estonia	5.53	7.01	-3.80	Senegal	-2.24	-2.10	1.96
Ethiopia	2.95	3.60	-2.17	Serbia	-2.65	-2.60	-1.56
FYR Macedonia	11.97	19.47	-7.63	Slovak Rep.	2.08	2.12	-1.98
Ghana	-5.12	-4.27	1.00	Slovenia	5.02	5.68	-2.82
Guatemala	-5.36	-4.22	7.37	South Africa	-5.23	-5.21	-2.94
Honduras	11.12	16.85	-7.77	Sri Lanka	-11.22	-6.56	3.20
Hungary	1.85	1.93	-1.65	Sweden	3.93	4.40	-2.79
India	9.07	11.11	-6.31	Syrian Arab Rep.	-5.13	-4.93	0.70
Indonesia	10.55	14.13	-6.98	Tanzania	-4.53	-4.27	2.36
Ireland	3.69	4.72	-2.75	Thailand	10.94	13.77	-7.55
Jordan	7.03	8.97	-4.79	Tunisia	19.36	33.15	-13.18
Kazakhstan	-3.45	-3.17	0.47	Turkey	2.31	2.80	-1.17
Kenya	-4.90	-3.96	6.40	Uganda	-2.11	-1.71	3.07
Korea, Rep.	1.25	1.18	-1.37	Ukraine	-6.04	-5.66	1.74
Kyrgyz Rep.	0.77	0.75	-0.85	Uruguay	6.11	7.07	-4.85
Lao PDR	5.25	6.14	-3.72	Uzbekistan	-0.02	-0.03	-0.02
Latvia	7.76	10.60	-5.44	Vietnam	25.02	30.95	-18.15
Lebanon	0.68	0.67	-0.17	Zambia	-5.29	-3.61	3.21

The table reports the Vuong (1989) test statistic for different pairs of models: lognormal ($\mathcal{LN}$), gamma (Γ) and Fréchet. A positive value greater than 2.58 (the critical value at the 1% level) implies that Model 1 fits the data better than Model 2. A negative value below 2.58 implies the opposite. Otherwise we cannot discriminate between the two models.

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Appendix

A Proofs

A.1 Proof of Proposition 1.

Since export intensity E can be written as $E = Z/(1+Z)$, with $Z \equiv Z_x/Z_d$, it follows, from using the method of transformations for random variables, that the pdf of export intensity can be written as

$$h(e) = \frac{1}{(1-e)^2} \cdot \widetilde{f}\left(\frac{e}{1-e}\right), \quad e \in (0,1), \tag{A.1}$$

where $\widetilde{f}\left(\frac{z_x}{z_d}\right)$ denotes the pdf of the ratio of export to domestic revenue shifters.

A.1.1 Gamma

Coelho and Mexia (2007) show that the pdf of the ratio of two gamma-distributed random variables with parameters (α, s_x) and (α, s_d) (all strictly positive) respectively, is:

$$\widetilde{f}(z) = \frac{\left(\frac{s_d}{s_x}\right)^{\alpha}}{\mathcal{B}(\alpha,\alpha)} \left[1 + \left(\frac{s_d}{s_x}\right) z\right]^{-2\alpha} z^{\alpha-1}, \quad z > 0, \tag{A.2}$$

where $\mathcal{B}(\cdot,\cdot)$ is the Beta function.

Substituting (A.2) into (A.1), we obtain:

$$h^{\gamma}(e) = \frac{\left(\frac{s_d}{s_x}\right)^{\alpha}}{\mathcal{B}(\alpha,\alpha)} \times \frac{e^{\alpha-1}(1-e)^{-(1+\alpha)}}{\left[1 + \left(\frac{s_d}{s_x}\right)\left(\frac{e}{1-e}\right)\right]^{2\alpha}}, \quad e \in (0,1). \tag{A.3}$$

A.1.2 Fréchet

Nadarajah and Kotz (2006) show that the cdf of the ratio of two Fréchet distributions with parameters (α, s_x) and (α, s_d) (all strictly positive), is:

$$\widetilde{F}(z) = \frac{\left[\left(\frac{s_d}{s_x}\right) z\right]^{\alpha}}{1 + \left[\left(\frac{s_d}{s_x}\right) z\right]^{\alpha}}, \quad z > 0. \tag{A.4}$$

The cdf for export intensity is:

$$\begin{aligned} H^{\text{Fréchet}}(e) \equiv \text{Prob}[E \leqslant e] &= \text{Prob}\left[\frac{Z_x}{Z_d} \leqslant \frac{e}{1-e}\right], \\ &= \widetilde{F}\left(\frac{e}{1-e}\right), \end{aligned}$$

$$\frac{\left[\left(\frac{s_d}{s_x}\right)\left(\frac{e}{1-e}\right)\right]^{\alpha}}{1 + \left[\left(\frac{s_d}{s_x}\right)\left(\frac{e}{1-e}\right)\right]^{\alpha}}, \quad e \in (0,1). \tag{A.5}$$

Taking the derivative of (A.5) yields the pdf of export intensity:

$$h^{\text{Fréchet}}(e) = \alpha \left(\frac{s_d}{s_x}\right)^{\alpha} \times \frac{e^{\alpha-1}(1-e)^{-(1+\alpha)}}{\left[1+\left(\frac{s_d}{s_x}\right)^{\alpha}\left(\frac{e}{1-e}\right)^{\alpha}\right]^2}, \quad e \in (0,1), \tag{A.6}$$

A.2 Proof of Proposition 2.

A.2.1 Gamma

Rewrite (A.3) as:

$$h^{\gamma}(e) = \frac{(s_d s_x)^{\alpha}}{\mathcal{B}(\alpha,\alpha)} \times \frac{e^{\alpha-1}(1-e)^{\alpha-1}}{[s_x(1-e)+s_d e]^{2\alpha}}. \tag{A.7}$$

Thus, it follows that when $\alpha < 1$,

$$\lim_{e\to 0} h^{\gamma}(e) = \lim_{e\to 1} h^{\gamma}(e) \to +\infty, \tag{A.8}$$

which proves that the distribution (A.7) has modes at 0 and 1. We then need to verify that when $\alpha < 1$, $h^{\gamma}(e)$ has no additional modes; i.e. that $h^{\gamma}(e)$ does not have any local maxima in the interior of the support.

We can find the critical points of (A.7) by taking the derivative with respect to e and setting it equal to zero:

$$\frac{\mathrm{d}h^{\gamma}(e)}{\mathrm{d}e} = \frac{e^{\alpha-2}\left[\left(\frac{s_d}{s_x}\right)e+(1-e)\right]^{-(2\alpha+1)}\left[\frac{s_d}{s_x}(1-e)\right]^{\alpha}}{(e-1)^2\mathcal{B}(\alpha,\alpha)} \times \left(Ae^2+Be+C\right) = 0, \tag{A.9}$$

where $A = 2\left(s_d/s_x - 1\right)$, $B = (3-\alpha)-(s_d/s_x)(1+\alpha)$ and $C = \alpha - 1$.

We use the intermediate value theorem to show that only one of the roots of the quadratic polynomial $\mathcal{P}(e) = Ae^2+Be+C$ lies in the interval $(0,1)$, by showing that $\mathcal{P}(0)\cdot\mathcal{P}(1) < 0$ when $\alpha < 1$:

$$\mathcal{P}(0) = C = \alpha - 1.$$
$$\mathcal{P}(1) = A+B+C = 2\left(s_d/s_x-1\right)+(3-\alpha)-(s_d/s_x)(1+\alpha)+\alpha-1 = (s_d/s_x)(1-\alpha).$$

Since $h^{\gamma}(e)$ is continuous and has two asymptotes at 0 and 1 when $\alpha < 1$, it follows that the critical value in the interior of the interval $(0,1)$ has to be a minimum. This shows that when $\alpha < 1$, the distribution of export intensity is bimodal.

When $\alpha > 1$, we have $h^{\gamma}(0) = h^{\gamma}(1) = 0$, and still only one critical point in the interior of the support, which shows that the distribution of export intensity is unimodal.

When $\alpha = 1$, $h^{\gamma}(0) = s_d/s_x$ and $h^{\gamma}(1) = s_x/s_d$, since $\mathcal{B}(1,1) = 1$. Moreover, since

$$\frac{\mathrm{d}h^{\gamma}(e)}{\mathrm{d}e} = \frac{(s_d s_x)(s_x - s_d)}{[s_x(1-e)+s_d e]^3}, \tag{A.10}$$

there is a mode at 1 when $s_d/s_x < 1$ because the pdf is strictly increasing; conversely, when $s_d/s_x > 1$ the unique mode is at 0. When $s_d/s_x = 1$, then the distribution of export intensity becomes the uniform distribution, which is considered unimodal.

The pdf $h^{\gamma}(e)$ is skewed to the left when $s_d/s_x > 1$ (i.e. that for any $e \in (0,1)$, $h^{\gamma}(e) > h^{\gamma}(1-e)$), and therefore, that when $\alpha < 1$, the major mode is located at 0. Conversely, when $s_d/s_x < 1$, the pdf is right-skewed, which means that the major mode is located at 1. When $s_d/s_x = 1$, $h^{\gamma}(e) = h^{\gamma}(1-e)$, so the pdf is symmetric around 0.5.

A.2.2 Fréchet

We rewrite (A.6) as:

$$h^{\text{Fréchet}}(e) = \alpha\,(s_d s_x)^{\alpha} \times \frac{e^{\alpha-1}(1-e)^{\alpha-1}}{\left[s_x^{\alpha}(1-e)^{\alpha} + s_d^{\alpha}e^{\alpha}\right]^2}, \quad e \in (0,1). \tag{A.11}$$

Thus, it follows that

$$\lim_{e\to 0} h^{\text{Fréchet}}(e) = \lim_{e\to 1} h^{\text{Fréchet}}(e) \to +\infty, \tag{A.12}$$

when $\alpha < 1$. Which proves that the distribution (A.11) has modes at 0 and 1.

Unlike in the case of gamma-distributed revenue shifters, we cannot prove analytically that there is a unique critical point for the pdf (A.6) in the interior of the support. Instead, we solve numerically the non-linear equation $\frac{dh^{\text{Fréchet}}(e)}{de} = 0$ over a 100×100 grid for the shape and scale parameter. For each pair of parameters we solve the first-order condition using 500 different starting values in the interval $(0,1)$. We find that when $\alpha < 1$, we always converge to the same solution (a minimum), regardless of the starting value. This suggests that $h^{\text{Fréchet}}(e)$ has no additional modes other than 0 and 1.

A.3 Proof of Proposition 3.

A.3.1 Gamma

We use the result that the ratio of two independent gamma-distributed random variables $Z_d \sim \Gamma(\alpha, s_d)$ and $Z_x \sim \Gamma(\alpha, s_x)$ can be expressed in terms of the F distribution (Johnson et al., 1995). Namely,

$$\frac{\alpha s_d}{\alpha s_x} \cdot \frac{Z_x}{Z_d} \sim F(2\alpha, 2\alpha). \tag{A.13}$$

Since the median of a random variable distributed F with the same number of degrees of freedom in the numerator and denominator is 1, it follows from (A.13) that the median of the ratio Z_x/Z_d, z^{med}, is equal to s_d/s_x. Since the median of a monotone transformation of a random variable is equal to the transformation of the median, then $e^{med} = \frac{s_x}{s_d+s_x}$.

A.3.2 Fréchet

Using (A.5), we can easily find the median export intensity by solving the equation $H^{\text{Fréchet}}(e^{med}) = 0.5$:

$$\frac{\left[\left(\frac{s_d}{s_x}\right)\left(\frac{e^{med}}{1-e^{med}}\right)\right]^{\alpha}}{1 + \left[\left(\frac{s_d}{s_x}\right)\left(\frac{e^{med}}{1-e^{med}}\right)\right]^{\alpha}} = \frac{1}{2}, \tag{A.14}$$

which results in $e^{med} = \frac{s_x}{s_d+s_x}$.